MCS-033

MCA (Revised)

Term-End Examination

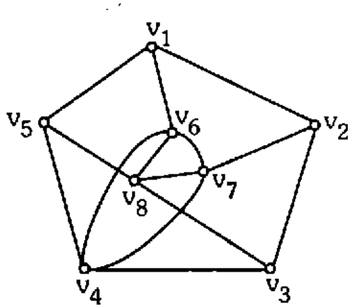
June, 2008

MCS-033 : ADVANCED DISCRETE MATHEMATICS

Time : 2 hours *Maximum Marks : 50*

Note : *Question no. 1 is* ***compulsory****. Attempt any* ***three*** *questions from the rest.*

1. (a) Consider the graph below :



(i) Find $\delta(G)$ and $\Delta(G)$.

(ii) Is the graph bipartite ?

(iii) Give a spanning tree of the graph.

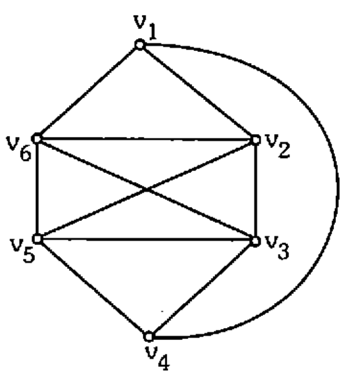
(iv) Draw the subgraph induced by the set $\{v_1, v_6, v_4, v_7, v_2\}$. 4

(b) Let a_n be the number of non-empty subsets of the set $\{1, 2, ..., n\}$. Prove the relation $a_n = 2a_{n-1} + 1$. 3

(c) State Euler's formula for a planar graph. Give an example of a planar graph with 5 vertices and 5 regions and verify Euler's formula for your example. 4

(d) In the country Marigynia, the currency is available in denominations of 1, 2 and 6 only. Find the generating function for the number of ways in which n currency units can be paid. 3

(e) Consider the following graph :



Is there an Eulerian circuit in this graph ? Is the graph edge traceable ? If yes, give an open trail containing all the edges. 3

(f) Give the order and degree of the following recurrences :

(i) $a_n^2 + na_{n-1} = 2^n$

(ii) $na_n + (n + 1) a_{n-1} + ... + (2n - 1) a_1 = 2^n$

(iii) $a_n = \sin^2 a_{n-1} + \sin^2 a_{n-2} + ... + \sin^2 a_0$ 3

2. (a) Define three-peg Tower of Hanoi problem and let T_n be the minimum number of moves that will transfer n disks from one peg to another peg. Show that $T_n = 2T_{n-1} + 1$. 5

(b) Draw $K_{3,3}$ and label its vertices. Give a path of length 5 in $K_{3,3}$. Give a path of length 6 if it exists. If you think such a path doesn't exist, give reasons for your answer. Give a family of graphs with n vertices, $n \geq 2$, which have a path of length $n - 1$. 5

3. (a) Let a_n be the number of comparisons required to arrange a list of n integers in decreasing order. Find a recurrence for a_n. Give the values of a_1 and a_2. 4

(b) An airline operates flights between 9 cities. What is the minimum number of flights the airline should operate so that it is possible to travel by its flights between any two cities, changing flights if necessary ? Solve the problem using graph theory. 4

(c) Draw a 4-regular graph on 6 vertices. 2

4. (a) Solve the recurrence

$$6a_n - 7a_{n-1} + a_{n-3} = 4, \; n \geq 3$$

if $a_0 = 14$, $a_1 = 1$, $a_2 = 5$. 7

(b) Give a planar drawing of $K_{2,4}$. Verify Euler's formula for $K_{2,4}$. 3

5. Solve the following recurrence using generating function : *10*

$$a_n + 2a_{n-1} - 15a_{n-2} = 2^n, \quad n \geq 2, \; a_0 = a_1 = 1$$