Teachers Teaching with Technology

T³ Scotland

T³ EUROPE

Recurrence Relations

RECURRENCE RELATIONS

Aim

The aim of this unit is to investigate linear recurrence relations of the form:

$$u_n = a\,u_{n-1} + b \qquad \text{or} \qquad u_{n+1} = a\,u_n + b$$

Objectives

Mathematical objectives

By the end of this session you should be able to:

- to find any term in a sequence given a general term u_n.
- be able to find the first few terms of a recurrence relation
- be able to find numerically or graphically the limit of a given recurrence relation

Calculator objectives

By the end of this session you should be able to:

- use of repeated calculation on home screen using [2nd][ANS].
- use of built in recurrence relation function in calculator.
- use of sequential graphing
- use of table to extract values in a given recurrence relation.

Evaluating Recurrence Relations

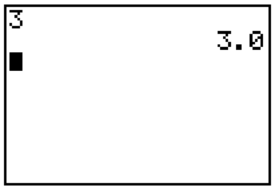
The basic principle of a recurrence relation is that the current answer is based on the previous answer which is based on the answer before and so on.
This principle is easily demonstrated on the TI.

Example 1

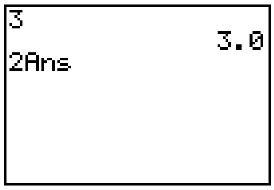
Given the recurrence relation $u_n = 2u_{n-1}$ find the first 6 terms when the first term is 3.

Calculator solution

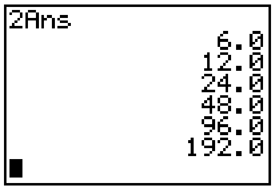
First clear the home screen [CLEAR].
Now input the first term to the memory by pressing [3][ENTER] to give the screen shown



Set up the recurrence relation using [2][2nd][ANS] as shown.



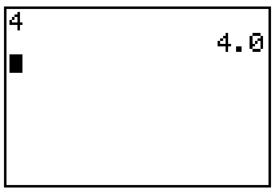
Now by repeated pressing of [ENTER] we get the terms shown (remembering to keep a count- term 1, term 2 etc).



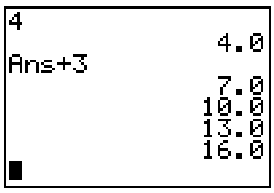
Example 2
Find successive terms in the recurrence relation $u_n = u_{n-1} + 3$ with $u_1 = 4$

Calculator solution

Enter the first term, [4][ENTER].



Set up the recurrence relation [2nd][ANS][+][3] and with repeated [ENTER] obtain the solution for as many terms as required.



Exercise:

For each of the following, find u_1, u_2, u_3, u_4, and u_{10}.

1. $u_n = 2\,u_{n-1} + 2$ $\qquad u_0 = 4$

2. $u_n = 0.5\,u_{n-1} + 4$ $\qquad u_0 = 2$

3. $u_n = 2.5\,u_{n-1} - 3$ $\qquad u_0 = 15$

4. $u_n = 0.2\,u_{n-1} - 6$ $\qquad u_0 = 28$

5. $u_n = 3\,u_{n-1} + 8$ $\qquad u_0 = 0.4$

6. $u_n = 1.6\,u_{n-1} + 4$ $\qquad u_0 = 1.9$

7. $u_n = 2.5\,u_{n-1} + 112$ $\qquad u_0 = 112$

8. $u_n = 0.1\,u_{n-1} - 0.01$ $\qquad u_0 = 16$

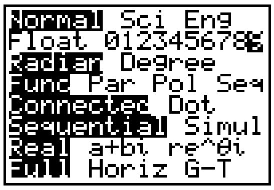
Limits

Using the calculator and the recurrence relation shown press enter to obtain the 25th term

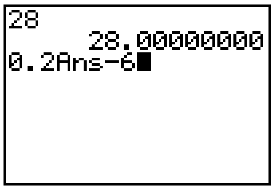
$u_{n+1} = 0.2\ u_n - 6 \qquad\qquad u_0 = 28$

Initially set your calculator in the most ‘exact’ mode.

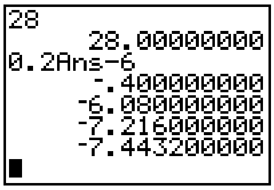
Press [MODE] and then use the cursor keys to highlight the number 9 on the second line of the display, this will force the TI-83 to round the answer and display the result to 9 decimal places.
Use [2nd][QUIT] or [CLEAR] to return to the home screen



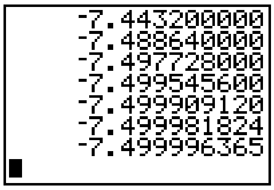
Enter the value of u_0. [28][ENTER].
Note: the display shows 9 decimal places.
Set up the recurrence relation. [0][.][2][ANS][-][6]



By repeatedly pressing [ENTER] the screen should scroll down recalculating the new value.

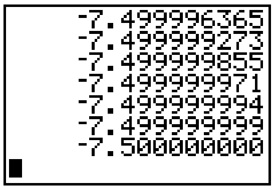


Notice how the display of zeros fills up.

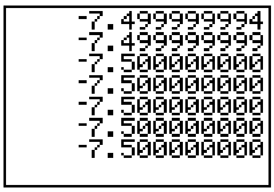


Once the display has filled with it jumps to -7.5000000

Then there is no change.



Why does this happen ?



EXERCISE:
Investigate the following recurrence relations and tick the boxes if you find a limit and enter the numerical value in the box shown

Recurrence relation	Initial value	Limit: Yes	Limit: No	Numerical value
1. $u_n = 2\,u_{n-1} + 2$	$u_0 = 4$	Yes	No	
2. $u_n = 0.5\,u_{n-1} + 4$	$u_0 = 2$	Yes	No	
3. $u_n = 2.5\,u_{n-1} - 3$	$u_0 = 15$	Yes	No	
4. $u_n = 0.2\,u_{n-1} - 6$	$u_0 = 28$	Yes	No	
5. $u_n = 3\,u_{n-1} + 8$	$u_0 = 0.4$	Yes	No	
6. $u_n = 1.6\,u_{n-1} + 4$	$u_0 = 1.9$	Yes	No	
7. $u_n = 2.5\,u_{n-1} + 112$	$u_0 = 112$	Yes	No	
8. $u_n = 0.1\,u_{n-1} - 0.01$	$u_0 = 16$	Yes	No	

Can you spot a pattern with the coefficients of u_n and whether or not a limit exists?
Make a statement about what you have found.

Make a conjecture about the value(s) you think that the coefficient must be in order for a limit to exist.

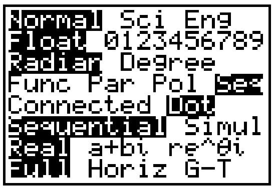
Graphical representation of recurrence relations

Example

A small forest contains 4000 trees.
Under a new forest management plan each year 20% of the trees will be harvested and 1000 new trees will be planted.
Investigate the long term state of the forest.

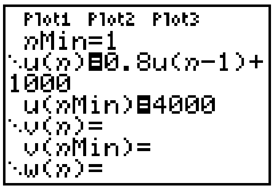
This yields the recurrence relation $u_{n+1} = 0.8\, u_n + 1000 \quad u_0 = 4000$

On the [MODE] screen set the TI in Sequence graphing mode (4th line), and dot option for graph style (5th line).



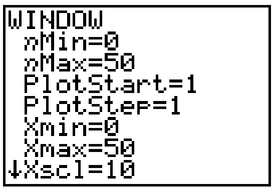
Press [Y=]to get to the recurrence relation setup screen.
Notice that this looks different than usual because of the changes made on the MODE screen.
[CLEAR] anything that may be shown.

Set *n*Min to 1 since we will calculate our first value after 1 year.
Input the recurrence relation in position u(*n*)=
Press [0][.][8] then [2nd][7], to get the u,
now [(][x,t,θ,n], to get n, then[-][1][)][+][1][0][0][0]



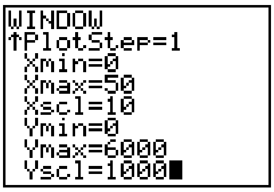
Input the initial value of 4000 in position u(*n*Min)=

Since we are going to graph we must now set the window range.
Press [WINDOW] to get the screen shown.



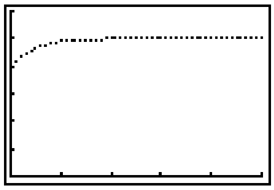
*n*Min defines the start of our time sequence and *n*Max the end.

Plotstart and Plotstep define where the plot is to start and what step size to take in our case start at year 1 and go in 1 year steps.



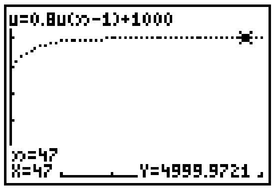
The *x* and *y* ranges are set as normal. Think about the numbers involved!!
This is shown on the two screens here.

Pressing [GRAPH] should give this display.



It can be seen that this graph increases and then levels out to a “horizontal” line.

This “horizontal” line is the limit.
Using the [TRACE] button and cursor keys the limit can be obtained.



From the graph it can be seen that the recurrence relation,

$u_{n+1} = 0.8\, u_n + 100$ has a limit = ________ when $u_0 = 4000$

On the [Y=] screen, try altering the value of u_0 .

What happens to the limit ?

____

____

____

____

____

Exercise

On the TI-83 obtain the graph of these recurrence relations and by a graphical method obtain the limit, if one exists.

1. $u_{n+1} = 1.2\, u_n + 40$ $\quad u_0 = 25$

2. $u_{n+1} = 0.2\, u_n + 120$ $\quad u_0 = 560$

3. $u_{n+1} = 0.8\, u_n - 17$ $\quad u_0 = 23$

4. $u_{n+1} = -0.6\, u_n + 52$ $\quad u_0 = 1000$